A GENERALIZED MULTIPLEXER THEORY AND DESIGN OF MANIFOLD MULTIPLEXERS

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ABSTRACT

A general direct analytical design procedure is presented for multiplexers having any number of channels, with arbitrary channel complexity, bandwidths, and inter-channel spacings. Approximate formulas developed for directly-connected filters have limitations which can be improved by normal immittance compensation. However the approximations are greatly improved by spacing the filters along a manifold in a way which not only separates the filters physically but also utilizes the inter-channel phase shifters to provide the immittance compensation. Contiguous cases are designable by the theory, which is presently applicable to narrow bandwidth channels. The theory has been proven both by computer simulation and by a limited number of practical designs.

Introduction

In two previous papers,direct design formulas were presented for bandpass channel diplexers.[1,2] The extension of the procedure to the general multiplexer case may be developed in two distinct phases. In the first design formulas are derived for interacting channel filters having direct connection without immittance compensation networks. This is an important practical configuration,and gives acceptable results for a wide variety of specifications, as demonstrated by computer analysis and by experimental results. The main limitation is that the channels may not be spaced too closely in frequency.

In the second phase of the theory,consideration is given to the design of immittance compensation networks. Significant improvement in performance may be obtained by addition of "dummy channels" to compensate the lowest and highest frequency channels of the multiplexer, which are always mismatched the worst. However a superior technique has been developed which utilizes a manifold as the immittance compensation network, also serving to separate the filters physically. This extended theory may be applied even to the limiting case of contiguous band coverage.

Theory of multiplexers with direct connection

As in the previous papers on diplexers[1,2] the theory commences from lumped element doubly terminated channel filters operating in isolation. Formulas are then developed which compensate for the interaction which takes place when the channels are connected.

There exists a large variety of lumped element doubly terminated lowpass prototype filters, ranging from the Chebyshev filter through to linear phase filters with finite attenuation poles. These normally have equiripple passband amplitude response with the maximum number of ripples. Thus,there is perfect transmission at n points $\omega=\omega_i$, $i=1\rightarrow n$. Defining the set of numbers ω_i and the level of the equiripple behaviour uniquely defines the filter even for elliptic function or linear phase filters. Furthermore, the set of numbers ω_i represents a very sensitive description of the transfer function in the sense that small variations will cause significant changes in the transfer function response. Thus, any modification to a filter due to interactions with any additional circuit elements must tend to preserve this set of frequencues ω_i,and this is the design principle used here.

The low pass prototype filter is assumed to have the circuit format shown in Fig.1. Here J_k is the characteristic admittance of the inverter between the k^{th} and $(k+1)^{th}$ shunt capacitors. The low-pass frequency

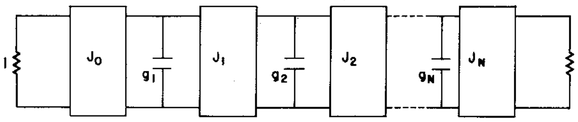

Fig.1 Low-pass prototype

domain is denoted by ω' with the cut-off frequency of the filter given by ω_1'. The insertion loss characteristics of the n-channel multiplexer,indicating the insertion losses from the common port to the n output ports,is shown in Fig.2.

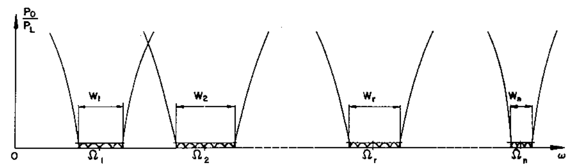

Fig.2 Insertion loss of general multiplexer

Here we have channel bandwidth W_r centered at frequency Ω_r, $r=1\rightarrow n$. The ripple levels of the channels are not required to be identical. The n-channel parallel-connected multiplexer is shown in Fig.3.

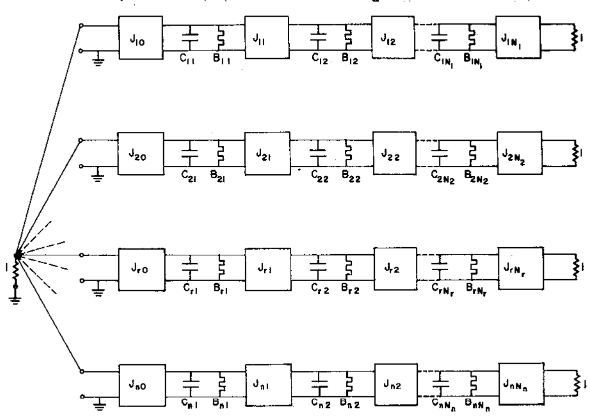

Fig.3 Parallel-connected multiplexer

The r^{th} channel is derived from the low pass prototype, using the frequency transformation

$$\omega' \rightarrow \frac{2\omega_1'}{W_r}(\omega-\Omega_r) \quad (1)$$

This changes each prototype shunt capacitor g_k into a capacitor C_{rk} in parallel with a frequency invariant susceptance B_{rk}, where

$$C_{rk}=\frac{2g_k\,\omega_1'}{W_r},\quad B_{rk}=-C_{rk}\Omega_r \quad (2),(3)$$

Now scale all center frequencies by a constant α, so that frequency transformation (1) is modified to

$$\omega' \to \frac{2\omega_1'}{W_r}(\omega-\Omega_r\alpha) \tag{4}$$

This modification maintains the same bandwidth for each channel, but changes the channel separations. Thus for α large all guard bands are large, so that the interaction between filters in a parallel connection is reduced. In fact as α tends to infinity the filters will not interact, and the individual passband performances will be maintained in the multiplexer. For each channel, an increase of 6 dB in the attenuation level over the passband regions of all other channels occurs, due to the potential-divider action resulting from the input admittance of the other channels being in parallel with that of the reference channel.

As α is decreased to the design requirement of unity, the channels will interact. The element values may then be expressed as a power series in α^{-1} with leading terms being the original doubly terminated prototype values. Following the reasoning given previously[1,2] where it is shown that the impedance inverters must be an even function of α, the frequency independent susceptances an odd function of α, and that the shunt capacitors may be retained unchanged, we have

$$J_{r0}^2 \to 1-\gamma_{r02}\alpha^{-2} \tag{5}$$

$$J_{r1}^2 \to J_{r1}^2(1-\gamma_{r12}\alpha^{-2}) \tag{6}$$

$$B_{r1} \to -C_{r1}(\Omega_r\alpha+\beta_{r11}\alpha^{-1}+\beta_{r13}\alpha^{-3}) \tag{7}$$

$$B_{r2} \to -C_{r2}(\Omega_r\alpha+\beta_{r23}\alpha^{-3}) \tag{8}$$

and all other elements are unchanged. In this approximation, terms of order α^{-4} and above will be neglected.

An expression for the input admittance to the common port of the multiplexer may now be derived. It is necessary to derive the "in-band" input admittance to the r^{th} channel and the "out-of-band" admittances to the other (n-1) channels within the frequency band of this r^{th} channel. The required common port input admittance is then the sum of all these admittances. A closed form expression for the in-band admittance as a power series in α^{-1} is derived using a key element in the theory, namely that the input admittance is unity at the frequencies of perfect transmission for the original filters, where $\alpha\to\infty$. The formula for the out-of-band admittance is derived in a straightforward manner. Summation of the expressions for the admittances gives a final expression of the form

$$\begin{aligned} Y_{in}(\Omega_r\alpha+\omega_i) &= Y_r(\Omega_r\alpha+\omega_i)+\sum_{m=1\neq r}^{n} Y_m(\Omega_r\alpha+\omega_i) \\ &= 1+A_{10}\alpha^{-1}+(A_{20}+A_{21}\omega_i)\alpha^{-2} \\ &\quad +(A_{30}+A_{31}\omega_i+A_{32}\omega_i^2)\alpha^{-3}+\epsilon(\alpha^{-4}) \end{aligned} \tag{9}$$

where Y_r is the input admittance of the r^{th} in-band channel, Y_m are the admittances of the out-of-band channels, and $(\Omega_r\alpha+\omega_i)$ is the i^{th} frequency of perfect transmission, the ω_i being undetermined variables. Hence at these frequencies we may set $Y_{in}=1$, giving a set of six equations for each channel obtained by setting the coefficients A_{ij} in (9) to zero. Unfortunately there are only five parameters per channel as seen in (5-8), so that these equations can not be satisfied exactly. A partial solution leaves a residual term in $\omega_i\alpha^{-3}$, and the following solution for parameters defined in (5-8):

Define $$S_\ell = \sum_{m=1\neq r}^{n} \frac{1}{C_{m1}(\Omega_r-\Omega_m)^\ell} \tag{10}$$

then $$\beta_{r11}=S_1/C_{r1}, \quad \gamma_{r12}=S_2/C_{r1} \tag{11),(12}$$

$$\gamma_{r02}=S_2/C_{r1}-S_1^2, \quad \beta_{r23}=J_{r1}^2 S_3/C_{r1}^2 C_{r2} \tag{13),(14}$$

$$\beta_{r13}=S_3/C_{r1}^3-S_1S_2/C_{r1}^2+S_1^3/C_{r1} + \frac{1}{C_{r1}}\sum_{m=1\neq r}^{n}\left(\frac{-\gamma_{m02}}{C_{m1}(\Omega_r-\Omega_m)}+\frac{\beta_{m11}}{C_{m1}(\Omega_r-\Omega_m)^2}+\frac{J_{m1}^2}{C_{m1}^2C_{m2}(\Omega_r-\Omega_m)^3}\right) \tag{15}$$

These formulas have been programmed, and applied to the design of practical multiplexers, as in the following.

<u>Practical Example: A 5-channel interdigital multiplexer</u>

Fig.4 shows the common port return loss of an X-band 5-channel interdigital multiplexer, consisting of filters all of degree 8, return loss ripple of 19.08 dB, and bandwidth of 400MHz with inter-channel spacings of 600MHz. The performance shown is for the actual distributed interdigital network. The solid line graph shows the performance assuming that the input impedance of all filters were to add, neglecting a small deterioration given by assuming a more exact equivalent circuit of the junction. The dotted line indicates the deterioration, mainly in the outer channels, which could easily be tuned out in practice.

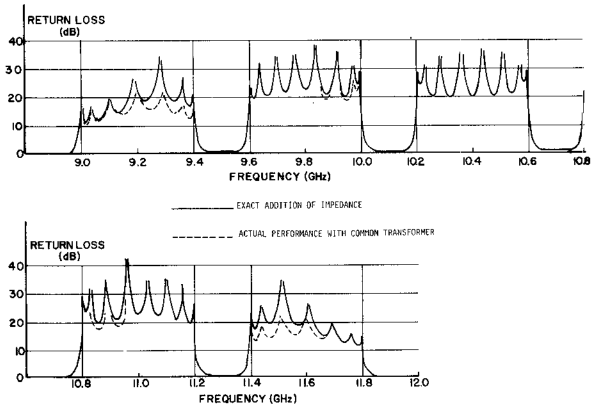

Fig.4 Return loss of interdigital multiplexer

This multiplexer was built in the form of a star, as shown in Fig.5. As expected, the outer channels proved to be somewhat more difficult to match than the inner three channels, but finally the worst return loss obtained was 15.5 dB (VSWR 1.4:1). The inter-channel isolation is greater than 60dB, and the insertion loss in the central 350MHz of each passband was less than 1.2dB, in good agreement with theory for interdigital filters having 0.25 in. ground plane spacing.

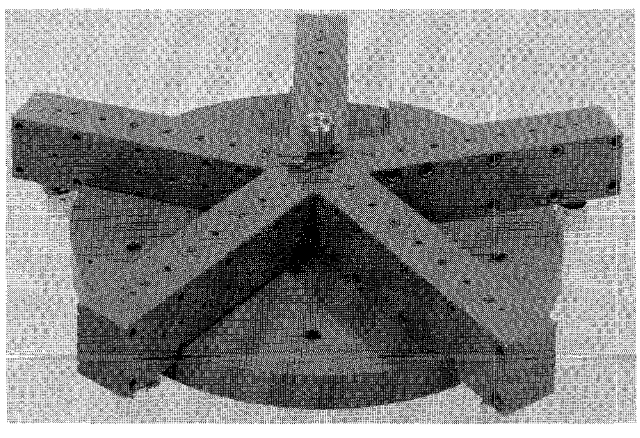

Fig.5 5-channel X-band multiplexer

Theory of Manifold multiplexers

The theory of the manifold multiplexer, shown in Fig.6, is similar to that outlined in the previous section, but Equ.(9) must be modified to take account of the phase shifters Θ_m. An initial simplified theory assumes that the phase shifters are frequency-independent. The dependence of Θ_m on α is given by

$$\Theta_m = \tan^{-1}(B_m \alpha^{-1}) \qquad (16)$$

where B_m is a constant. Note that as $\alpha \to \infty$ (the case of decoupled channels), $\Theta_m \to 0$ or an integral multiple of π, i.e. the channels are effectively connected in parallel. $\alpha=1$ gives the actual situation to be determined.

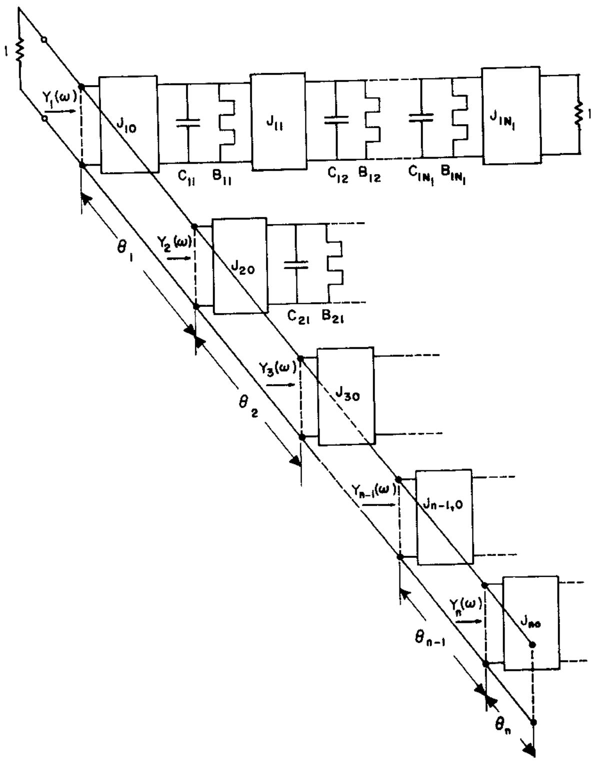

Fig.6 Manifold multiplexer

By connecting the filters to the manifold an extra (n-1)parameters $B_1, B_2, \ldots B_{n-1}$ have been obtained (the parameter B_n being redundant),enabling the improved theory to be carried out. Given the expressions for the in-band and out-of-band channel admittances as a power series in α^{-1},it is necessary to form the admittances at the junction of the in-band channel looking out in each direction in the manifold. The final equations are derived by forcing the condition of a conjugate match at this junction. In order for this to result in an explicit closed form solution these admittances must themselves be expressed in closed form. In most problems involving a general cascade of networks no such closed form solution exists,e.g.the transfer matrix is the product of the individual transfer matrices,and except in special cases,no further simplification to the transfer matrix product can be made. Fortunately the cascade of phase shifters alternated with the shunt admittances of the out-of-band channels is such a special case. It is necessary only to retain terms up to order α^{-3} in the transfer matrix products,which simplifies derivation of the results.

The conjugate match condition for an n-channel multiplexer consist of 6n equations with 6n-1 unknowns, as compared with 6n equations with 6(n-1)unknowns previously. In the case of a diplexer these have an exact solution,but for the triplexer and above only 5 of the 6 conditions may be satisfied in the n^{th} channel. Fortunately this causes negligible deterioration especially after further refinements are added utilizing a small degree of computer optimization.

The frequency dependent manifold

Incorporation of the frequency dependence of the manifold in a manner compatible with the established design procedure is by no means obvious. Defining ψ_m as the electrical length of the manifold from channel m to the reactive termination,then the final expression adopted for this electrical length is

$$\psi_m \to \tan^{-1}[\alpha^{-1}\tan \frac{\alpha^{-1}\psi_m \omega}{\Omega_m}] \qquad (17)$$

and in the previous theory Θ_m is replaced by $\psi_m - \psi_{m+1}$. The resulting rather complicated equations are readily expandable as a power series in α^{-1}, giving closed form expressions for the channel and manifold parameters. Additional slight computer optimization is performed to minimize the effect of the approximations.

The theory has been applied to the design of several multiplexers having either widely spaced or contiguous channels. As an example of the latter a 10-channel manifold was designed in WR75 for the 14-14.5 GHz band. The doubly-terminated prototype filters were selected to have 6 cavities with 22dB return loss bandwidth of 43MHz,and the channel center frequencies were spaced by 49MHz(giving 3dB crossovers). The multiplexer synthesized from the theory was analysed on the computer, and showed a worse return loss at the common port of 16 dB,this occurring near the edges of the passbands.

This multiplexer has not yet been built,but there is good reason to believe that practical versions would work in accordance with theory since the computer analysis is known to be accurate, e.g. in predicting the performance of simpler designs which have been realized in practice.

References

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2. J.L.Haine and J.D.Rhodes,"Direct Design Formulas for Asymmetric Bandpass Channel Diplexers", IEEE Trans.on Microwave Theory and Techniques,Vol.MTT-25, pp.807-813, October 1977.